HOW ROBUST ARE MIXING DIAGNOSTICS INFERRED FROM SATELLITE ALTIMETRY?

SHANE R. KEATING AND K. SHAFER SMITH

1. Introduction

Quantifying the mixing properties of the surface ocean is crucial to the understanding of observed temperature and chlorophyll distributions [1], pollutant dispersal [2], plankton dynamics [3], and for the development of skillful eddy parameterization schemes [4, 5]. On regional scales, in-situ observations of surface drifters and controlled tracer release experiments as well as satellite imagery of sea-surface temperature and ocean color provide valuable data on horizontal dispersion and mixing (see for example, [6, 7, 8, 9]). In addition, repeat-track satellite measurements of sea-surface height anomalies are now routinely used to estimate geostrophic velocities at the ocean surface, providing an unprecedented view of the global surface eddy field.

In recent years, a number of studies have made use of surface velocity data inferred from satellite altimetry to diagnose mixing and transport in the surface ocean on both regional and global scales [10, 11, 12, 13, 14, 15, 16, 17, 18]. This is typically done by using timeseries of the observed surface velocity field, interpolated onto a Mercator grid, to advect virtual tracers or particles, from which the relevant Eulerian or Lagrangian diagnostics can be calculated. Despite great leaps in the quality and resolution of satellite-altimetry data, however, the inferred velocity fields still suffer from inevitable gaps arising from nonuniform satellite tracks, tracking errors, cloud cover, and sea ice, effectively reducing the spatial resolution of the gridded velocity fields to roughly the deformation scale. Although mesoscale processes can be expected to be captured by this range, the effect of unresolved scales on eddy transport processes is difficult to quantify.

This point has been appreciated in the atmospheric sciences community for some time. Bennett [19] argued that the relative dispersion of balloons in the stratosphere depends sensitively on the slope of the kinetic energy spectrum and identified two dynamical regimes governing the evolution of advected quantities. For self-similar kinetic energy spectra of the form $E(k) \propto k^{-\alpha}$, where k is the horizontal wavenumber, particle dispersion will be governed by eddies on the energy-containing scale when $\alpha > 3$ ('spectrally nonlocal' dynamics), and by eddies on the scale of dispersion when $\alpha < 3$ ('spectrally local' dynamics). This gradual transition from local to non-local dynamics can be seen most clearly in the

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characteristic timescale associated with the horizontal wavenumber k [20, 21],

$$\tau(k) = \left(\int^k p^2 E(p) \mathrm{d}p \right)^{-1/2} . \qquad (1)$$

The timescale τ represents the local distortion time due to the effective mean shear from wavenumbers $p \leq k$ [22]. For steep kinetic energy spectra with $\alpha > 3$, the integral in (1) is dominated by the large scales (small p) and is effectively independent of the local wavenumber k. In the nonlocal case, then, particle dispersion is controlled by the velocity field at the largest scales, typically, the energy-containing scales. Conversely, for sufficiently shallow kinetic energy spectra with $\alpha < 3$, the timescale (1) is a function of the local horizontal wavenumber and particle dispersion on a given horizontal scale is controlled by the velocity field on that scale. For the marginal case of $\alpha = 3$, the timescale $\tau(k)$ depends only logarithmically on the horizontal wavenumber and particle dispersion is weakly nonlocal.

The distinction between regimes of local and non-local spectral dynamics places an important constraint on our ability to diagnose mixing from low-resolution velocity datasets. In particular, the paradigm of chaotic advection — with its intuitively appealing picture of chaotic mixing on small scales due to advection by smoothly varying, quasi-periodic velocity fields — is a special case of nonlocal dynamics in which the Eulerian correlation time is long compared with the Lagrangian correlation time [23]. Conversely, chaotic advection is inconsistent with local dynamics, suggesting that mixing in this regime is highly sensitive to the spatial and temporal resolution of the velocity field. As such, the reliability of diagnostics of mixing based upon low-resolution velocity fields is intrinsically linked to the steepness of the kinetic energy spectrum, a point made independently by Bartello [24] and Shepherd *et al.* [21] in the context of the atmosphere. Motivated by simulations of atmospheric turbulence exhibiting kinetic energy spectra with $\alpha > 3$, $\alpha \approx 3$ and $\alpha < 3$ in the stratosphere, extratropical troposphere and mesosphere, respectively, these authors concluded that in the stratosphere, relatively coarsely resolved winds can be used to accurately advect tracers and Lagrangian particles and that, consequently, offline diagnostics (tracer variance, correlation times, Lyapunov exponents) based upon the temporal evolution of these fields are fairly reliable measures of mixing in the stratosphere. Likewise, it was concluded that care must be taken in extending these diagnostics to winds in the mesosphere and, to a lesser extent, the extratropical tropopause, as they are likely to be resolution dependent.

Very few direct measurements of the surface ocean kinetic energy wavenumber spectrum exist. Stammer [12] analyzed midlatitude sea-surface height and surface velocity data from the TOPEX/Poseidon satellite and found a spectral exponent of $\alpha \approx 3$ over the mesoscale-submesoscale range, a result that is consistent with a picture of geostrophic turbulence driven by baroclinic instability [25, 26, 27, 28, 29, 30, 31]. However, calibration error and noise dominate the TOPEX/Poseidon signal on scales smaller than about 100 km so measurements of power-law exponents from this range should be viewed with caution. More recently, attention has focused on the role of surface trapped modes that are not captured by the conventional geostrophic picture of oceanic turbulence and act to flatten the kinetic energy spectrum at high wavenumbers. A growing body of evidence suggests that these modes are crucial to the understanding of submesoscale dynamics and transport in the upper ocean: these include drifter trajectories [7] and more recent satellite measurements [32, 33] as well as high-resolution primitive equation ocean models [34]. The results of the latter study suggests a significantly shallower ($\alpha \approx 2$) kinetic energy spectrum in the meso-submesoscale range. If this is indeed the case, it raises important questions about the reliability and robustness of altimetry-inferred mixing diagnostics in the surface ocean.

In this paper, we study a range of Eulerian, Lagrangian, and quasi-Langrangian mixing diagnostics in controlled numerical simulations of quasigeostrophic turbulence and probe their sensitivity to changes in sampling resolution. As such, our study is very much in the spirit of Bartello [24] and Shepherd *et al.* [21]; however, our emphasis here is on the relative strengths and weaknesses of those diagnostics that have actually been employed using satellite altimetry data.To evaluate the importance of surface trapped modes on resolution sensitivity, we employ two phenomenological models of quasigeostrophic turbulence: the classical two-layer Phillips model of geostrophic turbulence [35], and the nonlinear Eady model, in which the dynamics is driven entirely by temperature anomalies on upper and lower surfaces bounding a region of constant potential vorticity [36]. In each models, kinetic energy is injected at small wavenumber via baroclinic instability: indeed, we note that these models are simultaneously taken as a prototype for the linear baroclinic instability problem. However, the nonlinear (turbulent) dynamics of the two models are quite distinct, with the Eady model exhibiting a kinetic energy spectral exponent of $\alpha = 5/3$ at large wavenumber, in contrast to the well-known $\alpha = 3$ forward cascade of the two-layer Phillips model [36, 37, 38]. As such, the Phillips model and the nonlinear Eady model are complementary, if simplified, models of the real ocean exemplifying either interior- or surface-dominated dynamics, and serve as valuable testbeds in which to directly probe the sensitivity of altimetry-inferred diagnostics to the unresolved scales.

The paper proceeds as follows: in section 2 we describe the various diagnostics of mixing that are to be tested. In section 3 we outline and contrast the two-layer Phillips model and the nonlinear Eady model...

2. Diagnosing mixing from satellite altimetry

The advent of satellite altimetry, heralded by the launch of Seasat in 1978, provided for the first time a means of calculating global geostrophic currents at the ocean surface directly from the sea-surface elevation relative to the geoid. Modern altimetric datasets such as AVISO merge multiple altimeter measurements of the along-track sea-surface anomaly, corrected for tidal, topographic and atmospheric effects, and interpolate the results onto a Mercator grid. From this, one can estimate the geostrophic streamfunction, given by $\psi = gh/f$, where f is the Coriolis parameter, g is the gravitational acceleration, and h is the sea-surface fluctuation, and thence the surface velocity field.

3. Nonlinear Eady model

4. Results

5. Conclusions

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